

SOTGrps

Constructing and identifying groups of small order type

1.3

29 August 2024

Eileen Pan

Eileen Pan Email: xpan.eileen@gmail.com

Homepage: <https://xpan-eileen.github.io/about/>

Address: School of Mathematics Monash University
VIC 3800
Melbourne, Australia

Contents

1	The SOTGrps package	3
1.1	Main functions	3
	References	7
	Index	8

Chapter 1

The SOTGrps package

With some overlaps, the SOTGrps package extends the Small Group Library to give access to some more “small” orders. For example, it constructs a complete and irredundant list of isomorphism type representatives of the groups of order

- that factorises into at most four primes;
- p^4q , for distinct primes p and q .

The mathematical background for this package is described in [DEP22].

1.1 Main functions

In addition to the functions described below, the SOTGrps package also extends the the Small Groups Library as provided by the SmallGrp package: with SOTGrps loaded, functions such as NumberSmallGroups, SmallGroup or IdGroup will work for orders support by SOTGrps but not by SmallGrp.

Note: for orders support by SOTGrps *and* by SmallGrp, the respective ids as produced by IdGroup versus IdSOTGroup in general do not agree. In a future version we may provided functions to convert between them.

1.1.1 AllSOTGroups

▷ AllSOTGroups(n [, $filter$]) (function)

takes in a number n that factorises into at most four primes or is of the form p^4q (p, q are distinct primes), and returns a complete and duplicate-free list of isomorphism class representatives of the groups of order n . Solvable groups are using refined polycyclic presentations. By default, solvable groups are constructed in the filter IsPcGroup, but if the optional argument $filter$ is set to IsPcpGroup then the groups are constructed in that filter instead. Nonsolvable groups are always returned as permutation groups.

Example

```
gap> AllSOTGroups(60);  
[ <pc group of size 60 with 4 generators>, <pc group of size 60 with 4 generators>,  
  <pc group of size 60 with 4 generators>, <pc group of size 60 with 4 generators>,  
  <pc group of size 60 with 4 generators>, <pc group of size 60 with 4 generators>]
```

```
<pc group of size 60 with 4 generators>, <pc group of size 60 with 4 generators>,
<pc group of size 60 with 4 generators>, <pc group of size 60 with 4 generators>,
<pc group of size 60 with 4 generators>, <pc group of size 60 with 4 generators>,
Alt( [ 1 .. 5 ] ) ]
```

1.1.2 NumberOfSOTGroups

▷ `NumberOfSOTGroups( $n$ )` (function)

takes in a number n that factorises into at most four primes or of the form p^4q (p, q are distinct primes), and returns the number of isomorphism types of groups of order n .

Example

```
gap> NumberOfSOTGroups(2*3*5*7);
12
gap> NumberOfSOTGroups(2*3*5*7*11);
Error, Order 2310 is not supported by SOTGrps.
Please refer to the SOTGrps documentation for the list of supported orders.
```

1.1.3 SOTGroup

▷ `SOTGroup( $n, i[, arg]$ )` (function)

takes in a pair of numbers n, i , where n factorises into at most four primes or of the form p^4q (p, q are distinct primes), and returns the i -th group with respect to the ordering of the list `AllSOTGroups( $n$ )` without constructing all groups in the list. The option of constructing a `PcpGroup` is available for solvable groups.

Example

```
gap> SOTGroup(2*3*5*7, 1);
<pc group of size 210 with 4 generators>
```

If the input i exceeds the number of groups of order n , an error message is returned.

1.1.4 IdSOTGroup (for IsGroup)

▷ `IdSOTGroup( $G$ )` (attribute)

takes in a group of order n and determines the SOT library number of G ; that is, the function returns a pair $[n, i]$ where G is isomorphic to `SOTGroup( $n, i$ )`. Note that if the input group is a `PcpGroup`, this may result in slow runtime, as `IdSOTGroup` may compute the Centre and/or the FittingSubgroup, which is slow for `PcpGroups`.

1.1.5 IsIsomorphicSOTGroups

▷ `IsIsomorphicSOTGroups( $G, H$ )` (function)

determines whether two groups G, H are isomorphic. It is assumed that the input groups are available in the `SOTGrps` library.

Example

```
gap> G:=Image(IsomorphismPermGroup(SmallGroup(690,1)));;
gap> H:=Image(IsomorphismPcGroup(SmallGroup(690,1)));;
gap> IsIsomorphicSOTGroups(G,H);
true
```

1.1.6 IsSOTAvailable

▷ IsSOTAvailable(n) (function)

returns true if the order n is available in the SOTGrps library, and false otherwise.

1.1.7 SOTGroupsInformation

▷ SOTGroupsInformation(n) (function)

prints information on the groups of the specified order. Since there are some overlaps between the existing SmallGrps library and the SOTGrps library. In particular, SOTGrps may construct the groups in a different order and so generate a different group ID; we denote such IDs by SOT. If the order covered in SOTGrps library has no conflicts with the existing library, then such a flag is removed.

Example

```
gap> SOTGroupsInformation(2^2*3*19);

There are 15 groups of order 228.

The groups of order  $p^2qr$  are either solvable or isomorphic to  $\text{Alt}(5)$ .
The solvable groups are sorted by their Fitting subgroup.
SOT 1 - 2 are the nilpotent groups.
SOT 3 has Fitting subgroup of order 57.
SOT 4 - 7 have Fitting subgroup of order 76.
SOT 8 - 9 have Fitting subgroup of order 38.
SOT 10 - 15 have Fitting subgroup of order 114.

gap> SOTGroupsInformation(2662);

There are 15 groups of order 2662.

The groups of order  $p^3q$  are solvable by Burnside's  $pq$ -Theorem.
These groups are sorted by their Sylow subgroups.
1 - 3 are abelian.
4 - 5 are nonabelian nilpotent and have a normal Sylow 11-subgroup and a
normal Sylow 2-subgroup.
6 is non-nilpotent and has a normal Sylow 2-subgroup with Sylow
11-subgroup [ 1331, 1 ].
7 - 9 are non-nilpotent and have a normal Sylow 2-subgroup with Sylow
11-subgroup [ 1331, 2 ].
10 - 12 are non-nilpotent and have a normal Sylow 2-subgroup with Sylow
11-subgroup [ 1331, 5 ].
13 - 14 are non-nilpotent and have a normal Sylow 2-subgroup with Sylow
11-subgroup [ 1331, 3 ].
```

15 is non-nilpotent and has a normal Sylow 2-subgroup with Sylow 11-subgroup [1331, 4].

References

- [DEP22] Heiko Dietrich, Bettina Eick, and Xueyu Pan. Groups whose orders factorise into at most four primes. *J. Symbolic Comput.*, 108:23–40, 2022. [3](#)

Index

AllSOTGroups, [3](#)

IdSOTGroup

 for IsGroup, [4](#)

IsIsomorphicSOTGroups, [4](#)

IsSOTAvailable, [5](#)

NumberOfSOTGroups, [4](#)

SOTGroup, [4](#)

SOTGroupsInformation, [5](#)